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AUTHORS' CLOSURE

The authors would like to thank Salonen for his interest in the paper and his comments. The authors used the approximate formulas for the stability functions expressed in eqns (5a) and (5b) to circumvent the inconvenience of having to use different expressions for the functions depending on whether the axial force is tensile or compressive. Furthermore, if the axial force is small, the exact stability functions will be indeterminate and so series expressions must be used to avoid numerical difficulties during computations. Another approach to avoid using different expressions for the stability functions is given by Goto and Chen (1987). The expressions were given in the form of infinite series. [Figure and equation numbers continue from the original paper.]

$$s_1 = \frac{\frac{1}{3} + \sum_{n=1}^{\infty} \frac{2(n+1)}{(2n+3)!}(\pi^2\rho)^n}{s} \tag{27a}$$

$$s_2 = \frac{\frac{1}{6} + \sum_{n=1}^{\infty} \frac{1}{(2n+3)!}(\pi^2\rho)^n}{s} \tag{27b}$$

where

$$s = \frac{1}{12} + \sum_{n=1}^{\infty} \frac{2(n+1)}{(2n+4)!}(\pi^2\rho)^n. \tag{27c}$$

Salonen is correct in pointing out that the approximate expressions for the stability functions expressed in the eqns (5a) and (5b) should be used only if $|\rho| \leq 2$. This fact has been addressed by the authors in a separate paper (Lui and Chen, 1987). However, it should be pointed out that for ordinary frame structures subjected to normal loadings, the axial forces in the members rarely exceed that limit. In fact, yielding would probably occur long before the limit is reached. It is therefore, concluded that the approximate expressions will be sufficient for the present purpose.

Apparently, for elastic structures that undergo large displacement with very slender members, it will be advisable to use the more exact form of the tangent stiffness matrix given by Oran and by Salonen. Nevertheless, the main objective of the present paper is not to perform large displacement elastica type analysis. Rather, the primary goal here is to investigate the effect of joint flexibility on the behavior of realistic frameworks. It is felt that the use of a more complicated tangent stiffness matrix is not warranted. For comparison, a four-story one-bay frame was analyzed (Chen and Lui, 1987) using the current approach and compared to the results obtained by Kassimali (1983). As can be seen from Fig. 21, the two results are very comparable. In the analysis, the beams were modeled by two elements and the columns were modeled by one element. The numbers on the curve designate the sequence of plastic hinge formation as the applied loads are increased monotonically.

Indeed, the bowing functions b_1 and b_2 expressed in eqns (8a) and (8b) are positive quantities. Therefore, absolute values were used for the computations. For $|\rho| \leq 0.05$, series expressions for the functions

$$b_1 \approx \frac{1}{40} - \frac{\pi^2\rho}{2800} + \frac{\pi^4\rho^2}{168000} - \frac{37\pi^6\rho^3}{388080000} \tag{28a}$$

$$b_2 \approx \frac{1}{24} - \frac{\pi^2\rho}{720} + \frac{\pi^4\rho^2}{20160} - \frac{\pi^6\rho^3}{604800} \tag{28b}$$

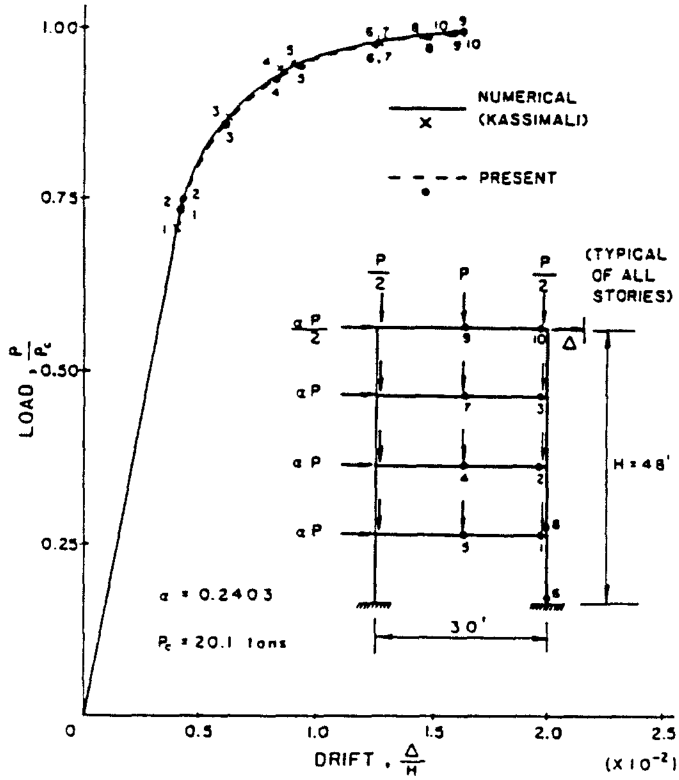

Fig. 21. Elastic plastic hinge analysis of a one-bay four-storey frame.

were used. p was taken as positive for a tensile axial force and negative for a compressive axial force.

It is interesting to note that if only one term is retained for b_1 and b_2 in eqns (28a) and (28b), the curvature shortening expressed in eqn (7) becomes

$$u_c = [\tfrac{1}{15}\theta_A^2 - \tfrac{1}{30}\theta_A\theta_B + \tfrac{1}{15}\theta_B^2]L. \tag{29}$$

Equation (29) was given by Jennings (1968) for which the curvature shortening effect was derived by assuming a cubic displacement field for the deflected shape of the member.

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